Mixed Non-Causal AR Processes and the Modelling of Explosive Bubbles

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9th French Econometrics Conference
November 30, 2017

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1. Random Coefficient Models

Consider the RCAR(1)

$$\begin{cases} X_t = \rho_t X_{t-1} + \eta_t, & \eta_t \sim iid\left(0, \sigma_\eta^2\right) \\ \rho_t = \exp\left(\phi + \lambda u_t\right) & u_t \sim NID\left(0, 1\right) \end{cases}$$

- if $\mathsf{E}\left[\max\left\{\log \eta_t^2, 0\right\}\right] < \infty$ and $\mathsf{E}\left[\max\left\{\log \rho_t^2, 0\right\}\right] < \infty$ then

  X_t admits strictly stationary causal solution $\Leftrightarrow \mathsf{E}\left[\log \rho_t^2\right] < 0 \therefore \phi < 0$

  X_t admits weakly stationary causal solution $\Leftrightarrow \log \mathsf{E}\left[\rho_t^2\right] < 0 \therefore \phi + \lambda^2 < 0$

  (Aue et al., 2006) with *strictly stationary* unit root $\mathsf{E}\left[X_t | X_{t-1}\right] = X_{t-1}$ if

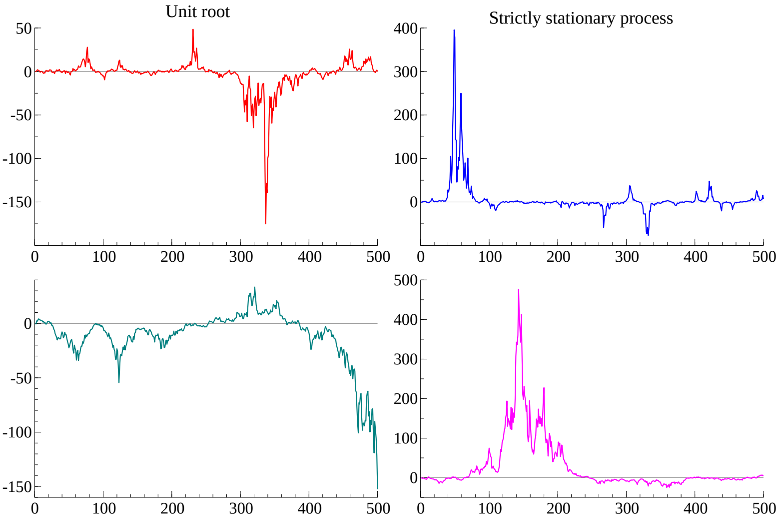
$$\log \mathsf{E}\left[\rho_t\right] = 0 \therefore \phi + \frac{\lambda^2}{2} = 0$$

  letting $\rho \sim \left(\rho, \sigma_\rho^2\right)$,

$$X_t = \rho X_{t-1} + \left(\left[\rho_t - \rho\right] X_{t-1} + \eta_t\right), \quad \text{with } \mathsf{Var}\left[X_t | X_{t-1}\right] = \sigma_\rho^2 X_{t-1}^2 + \sigma_\eta^2$$

  i.e., $X_t = \rho X_{t-1} + \epsilon_t \sqrt{\sigma_\rho^2 X_{t-1}^2 + \sigma_\eta^2}, \quad \epsilon_t$ mds with $\mathsf{E}\left[\epsilon_t^2 | \mathcal{F}_{t-1}\right] = 1$

Examples



Unit root
Strictly stationary process
50
0
-50
-100
-150
400
300
200
100
0
500
400
300
200
100
0
0
100
200
300
400
500

Link with Long Memory

- Baseline paths representation

$$X_t = \sum_{i=-\infty}^{t} \eta_i \prod_{j=i+1}^{t} \rho_j = \sum_{i=-\infty}^{t} \eta_i d_{i,t}$$

where $d_{i,t}$ and η_t is independent if η_t and ρ_t are. Hence, conditional on the realization of $\{u_t\}$, X_t is also a combination of baseline paths.

- Parke (1999) shows that if

$$X_t = \sum_{i=-\infty}^{t} \eta_i g_{i,t}$$

where $g_{i,t}$ is a random indicator function such that

$$g_{i,t} = 1\{t \leq i + n_i\} \quad \text{with} \quad P\left(g_{i,i+k} = 1\right) = p_k$$

e.g. if $\mathrm{P}(\rho_t = 0) \neq 0$

- Long memory arises if $\sum_0^j k p_k \to \infty$ and ARFIMA$(0, d, 0)$ if

$$p_{k+1} = \frac{k+d}{k+2-d} p_k$$

2. Inference

- Causal representation of MAR processes have lighter tails
  - how does it translate into inference?
- Anderson-Rubin statistic: consider the null $\mathrm{H}_0 : (\psi, \phi) = (\psi_0, \phi_0)$ in

$$(1 - \psi F)(1 - \phi B) X_t = (1 + \psi\phi) X_t - \psi X_{t+1} - \phi X_{t-1} = \varepsilon_t$$

and perform a test that $\mathrm{H}_0^* : c_1 = 0$ in auxiliary regression

$$\varepsilon_t^\kappa = c_0 + c_1 \varepsilon_{t-k}^\kappa + \zeta_t$$

where

$$\varepsilon_t^\kappa \underset{\mathrm{H}_0}{=} \left((1 + \psi_0\phi_0) X_t - \psi_0 X_{t+1} - \phi_0 X_{t-1} \right)^\kappa$$

i.e. as $T \to \infty$,

$$t_{c_1} \to 0$$

- Inference by grid search over non-rejected (ψ_0, ϕ_0) and estimation by $\min t_{c_1}^2$ (GMM-CUE)